

Differential Equation and Bifurcation Analysis

P.Priyanka¹, V.Padmavathi bai²

Assistant Professor, Department of Mathematics, Adhiyaman Arts and Science College for Women,
Uthangarai, India¹

Assistant Professor, Department of Mathematics, Adhiyaman Arts and Science College for Women,
Uthangarai, India²

ABSTRACT: The degree of a differential equation is the degree of the highest derivative which occurs in it, after the differential equation has been made free from radicals and fractions.

KEYWORDS: Prey model, Explicity, Implicity

I. INTRODUCTION

Bifurcation analysis is used to systematically detect unexpected dynamic behaviours of two predator-prey models that have recently received support and attention. Comparisons with other existing models are also made, in order to show that the singularities of the two examined models are due to some biological weakness of their functional forms. The predator-prey model is clear that there is a need for serious analysis of all of these models because many of them are somehow equivalent, while others are unreasonable from a biological point of view. In this predator-prey model, we consider in two cases. In the first case we keep only one model for equivalence class and in another class simply disregard the model. In the before analysis of poor model, is not easy to identify its weaknesses because (small insects) bugs be very difficult to find. Next step is to bugs can be find from the previous analysis. To select unexpected behaviours of model, here we use different approaches like persistence (continue to do the thing in which we have difficult). Another approach is bifurcation analysis. This model can have, for a given parameter setting, two alternative attractors (an equilibrium and a limit cycle).

This model has a very particular mode of behaviour, called tangle. Moreover, it is demonstrated that the tangle cannot be present if a particular pathology of the functional response is removed.

II. BIFURCATION ANALYSIS

DEGREE OF A DIFFERENTIAL EQUATION

The degree of a differential equation is the degree of the highest derivative which occurs in it, after the differential equation has been made free from radicals and fractions as far as the derivatives are concerned.

Example

$$Y \left(\frac{dy}{dx} \right) = \sqrt{x} \left(\frac{dy}{dx} \right)^2 + k$$

Which is of second degree.

LINEARLY DEPENDENT&INDEPENDENT SET OF FUNCTION

Consider n functions $y_1(x), y_2(x), \dots, y_n(x)$ are linearly dependent if there exist constants c_1, c_2, \dots, c_n (not all zero) such that

$$C_1 y_1 + C_2 y_2 + \dots + C_n y_n = 0$$

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If, however, identity implies that

$$c_1 = c_2 = \dots = c_n = 0.$$

Then y_1, y_2, \dots, y_n are said to be linearly independent.

EXPLICIT AND IMPLICIT

The solution is given as a function of the independent variable. In these cases, the solution is said to be explicit. In solving some differential equation an explicit solution cannot be determined. In these cases, the solution is said to be implicit.

SEPARABLE DIFFERENTIAL EQUATION

A differential equation that can be written in form $g(y) y' = f(x)$ is called a separable differential equation.

Rewriting,

$g(y) y' = f(x)$ in the form

$$g(y) \frac{dy}{dx} = f(x) \text{ yields}$$

$$g(y) dy = f(x) dx$$

So that,

$$\int g(y) dy = \int f(x) dx + c.$$

Ws

C is a constant.

III. BIFURCATION

The meaning of bifurcation is "The place where something divides into two branches". A differential equation system undergoes a bifurcation when a qualitative change in its orbit structure appearance, as one or more parameter of the dynamical system are changed. Bifurcation theory studies structurally unstable dynamics systems. The population dynamics by a conservation equation

$$\frac{dN(t)}{dt} = \text{births} - \text{deaths} + \text{migration}$$

HOPE BIFURCATION

The appearance or the disappearance of a periodic orbit through a local change in the stability properties of a steady point is known as the hope bifurcation.

LOGISTIC MODEL (Carrying Capacity)

The logistic model depends factors such as population size and density and rate of renew ability of its resources.

LIMIT CYCLE

A limit cycle on a plane or two Dimensional manifolds is a closed trajectory spirals into it either as time approaches infinity or as time approaches negative infinity.

SUPER CRITICAL AND SUB CRITICAL

If α is negative then there is a stable limit cycle for $\lambda > 0$,

$$z(t) = r e^{i\omega t}$$

Where $r = \sqrt{-\lambda / \alpha}$ and $\omega = \beta r^2$ this bifurcation is called super critical.

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If α is positive, then there is a unstable limit cycle for $\lambda < 0$. This bifurcation is called sub critical.

THE THREE TYPES OF FUNCTIONAL RESPONSE

The three general types of curves expected in various predator-prey situations.

Type I

- Type I is a linear relationship, where the predator is able to keep up with increasing density of prey by eating them in direct proportion to their abundance in the environment.
- If they eat 10% of the prey at low density, they continue to eat 10% of them at high densities. The dotted line indicates a maximum consumption rate that some authors attach to Type I foraging.

Type II

Type II describes a situation in which the number of prey consumed per predator initially rises quickly as the density of prey increases but then levels off with further increase in prey density.

Type III

Type III resembles Type II in having an upper limit to prey consumption, but differs in that the response of predators to prey is depressed at low prey density.

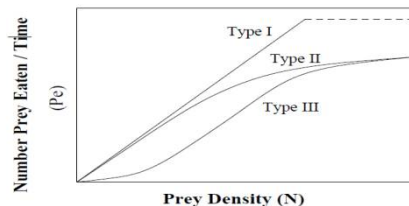


Figure: Three types of functional response relating prey density (N) and the number of prey eaten by one predator (Pe).

STABLE EQUILIBRIUM

When the centre of gravity of a body lies below point of suspension or support, the body is said to be in stable equilibrium. For example a book lying on a table is in stable equilibrium.

EXPLANATION

A book lying on a horizontal surface is an example of stable equilibrium. If the book is lifted from one edge and then allowed to fall, it will come back to its original position.

Other examples of stable equilibrium are bodies lying on the floor such as chair, table etc.

REASON OF STABILITY

When the book is lifted its center of gravity is raised. The line of action of weight passes through the base of the book. A torque due to weight of the book brings it back to the original position.

UNSTABLE EQUILIBRIUM

When the center of gravity of a body lies above the point of suspension or support, the body is said to be in unstable equilibrium

Example

Pencil standing on its point or a stick in vertically standing position.

EXPLANATION

If thin rod standing vertically is slightly disturbed from its position it will not come back to its original position.

This type of equilibrium is called unstable equilibrium, other example of unstable equilibrium are vertically standing cylinder and funnel etc.

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THE LESLIE-HOLLING MODEL

In the predator-prey model originally formulated by Leslie, the prey is logistic, the functional response of the predator is linear, and the predator is logistic, with carrying capacity proportional to prey density. This model is criticized and shown to be "not easily defensible" from a biological point of view. Here a slightly more complex model, called the Leslie Holling model (because the predator has a Holling type II functional response), is considered. Thus, the model is the following

$$\dot{x} = rx \left(1 - \frac{x}{K} \right) - \frac{ax}{b+x} y \quad (1.1)$$

$$\dot{y} = sy \left(1 - \frac{y}{hx} \right) \quad (1.2)$$

where x and y are indicators of abundance (large quantity) of prey and predator, and r, K, a, b, s and h are positive and constant parameters.

The Leslie-Holling models have one, and only one, strictly positive equilibrium for any parameter setting, but in the Leslie model this equilibrium is always asymptotically stable, so that there are no limit cycles, while in the Leslie-Holling model, the equilibrium can be unstable and surrounded by a stable limit cycle.

In this respect, the Leslie-Holling model seems to be similar to the most famous and often used predator-prey model

$$\dot{x} = rx \left(1 - \frac{x}{K} \right) - \frac{ax}{b+x} y \quad (1.3)$$

$$\dot{y} = e \frac{ax}{b+x} y - dy \quad (1.4)$$

sometimes called the Rosenzweig-McArthur model because its limit cycle is not stuck on the x, y axis as the limit cycle of Leslie-Holling model.

Leslie model is very often used by practitioners because logistic growth and type II functional response are well established notions. This model has an unexpected and intuitively unjustified dynamic behaviour for some parameter values. The general form of positive dynamical systems

$$\dot{y} = yg(x, y) \quad \dot{x} = xf(x, y)$$

and for any parameter setting there exists one, and only one, strictly positive equilibrium (x^+, y^+) given by,

$$x^+ = \frac{-(ahK - rK + rb) + \sqrt{(ahK - rK + rb)^2 + 4r^2Kb}}{2r} \quad (1.5)$$

$$y^+ = hx^+ \quad (1.6)$$

The Jacobian matrix J evaluated at (x^+, y^+) ,

i.e.,
$$J = \begin{bmatrix} x^+ f_x & x^+ f_y \\ y^+ g_x & y^+ g_y \end{bmatrix}$$

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can then be easily computed. By annihilating the trace of J , The explicit relationship among the parameters that characterizes a Hopf bifurcation can be found, provided the determinant of J is positive. Such a condition is

$$2rx^{+2} + x^+(rb - rK + sK) + sKb = 0 \quad (1.7)$$

Where x^+ is given by (1.5). Equations (1.5) and (1.7) allow the determination of Hopf bifurcation curves in any two-dimensional parameter space.

The Hopf bifurcation curve is split into two parts.

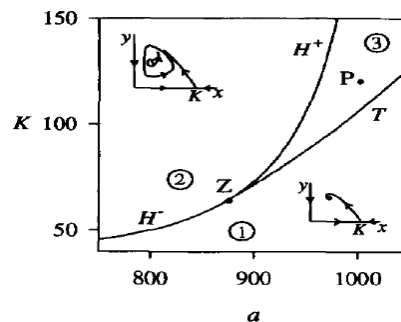


FIGURE 1. The Leslie-Holling model: (a) Hopf and tangent bifurcation curves in the parameter space (a, K) for $r = 5.4$, $s = 2.8$, $b = 2$, $h = 0.01$. (Specified by $a = 1000$, $K = 120$).

- The first, H^- , is a supercritical Hopf bifurcation crossing this curve from region 1 to region 2 the equilibrium (x^+, y^+) becomes unstable and surrounded by a stable limit cycle that is initially very small.
- The second, H^+ , is a subcritical Hopf bifurcation crossing this curve from region 2 to region 3, the unstable equilibrium (x^+, y^+) becomes stable and is surrounded by an unstable limit cycle.
- Thus, just below the curve H^+ in region 3, the system has two limit cycles around the stable equilibrium (x^+, y^+) , the internal cycle is unstable while the external cycle is stable.
- The bifurcation diagram points out the existence of an hysteresis (two discontinuities in the asymptotic behaviour of the system). In particular, if a is sufficiently high and K is very slowly and periodically varied over a large range, the two bifurcation curves T and H^+ are systematically crossed.
- Consequently, the system will undergo a catastrophic transition (from an equilibrium to a cycle) each time the curve H^+ is crossed from below, and a reverse catastrophic transition (from a cycle to an equilibrium) each time the curve T is crossed from above (hysteretic cycle).
- Trajectories starting inside the unstable limit cycle tend towards equilibrium, while trajectories starting outside tend towards the stable limit cycle.
- Since in region 1 the two cycles do not exist, there must be a line, rooted at point Z , on which the two cycles disappear. This is the tangent bifurcation curve T : coming from above, the two cycles collide on this curve and disappear.
- The existence of a zero Liapunov number, the possibility of having two distinct stable modes of behaviour is definitely unexpected.
- In fact, the model pretends to describe the simplest case of interactions between a prey and a predator under the assumption that these populations are fully isolated from the rest of the environment.

The Predator-Prey interactions is a widespread population process that has evolved many times within the metazoa. It can affect the distribution, abundance, and dynamics of species in ecosystems. Predator-prey interactions have an inherent tendency to fluctuate and show oscillatory behaviour.

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If predators are initially rare, then the size of the prey population can increase. As prey population size increases, the predator populations also begins to increase, which in turn has a detrimental effect on the prey population leading to a decline in prey numbers. As prey become scarce then the predator population size declines and the cycle starts again

The last case, multiple attractors can exist even if the populations are modelled as time-invariant second order systems, like food chains with a constant superpredator population or predator-prey models in which the prey is exposed to an extra predator, considered as a constant parameter. The theory of predator-prey interactions has made significant advances over the past few decades. The integration of theory and data remains a challenge in ecology.

Modern methods of analysis of behavioural, population, and metapopulation data are making it much easier to parameterize ecological models with ecological data, and this is opening the way for a closer integration of theoretical and empirical ecology.

The behaviour of the Leslie-Holling model is not in agreement with our understanding of the assumptions, and this pathology can be considered as the ultimate consequence of the absurdity already pointed out (a predator living forever on arbitrarily small amounts of resource).

Some authors do not recognize this fact as a weakness. The model is particularly suited for representing transitions between different stable modes of behaviour, a fact which is often present in ecosystems but which can correctly be explained only by more complex models.

IV. CONCLUSION

Bifurcation analysis can be systematically used to discover weaknesses of a predator-prey model. The method has been applied to two models of isolated predator-prey communities Leslie-Holling equations (1.1) (1.2) and Hassel Varley equations (2.1) (2.2), which have received, even recently, support and attention.

of both prey and predator in the second (Figure 4) . This agrees completely with the criticisms recently raised in to these and analogous models.

This suggestion should be followed, in particular, when the model is used to derive general theories or to build up larger models (e.g., food chains), or to interpret the role of seasons in predator-prey communities. Similar comparisons have also been performed for the Hassel-Varley Model.

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